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HEXAGONAL PANELS SUPPORTED AT DISCRETE
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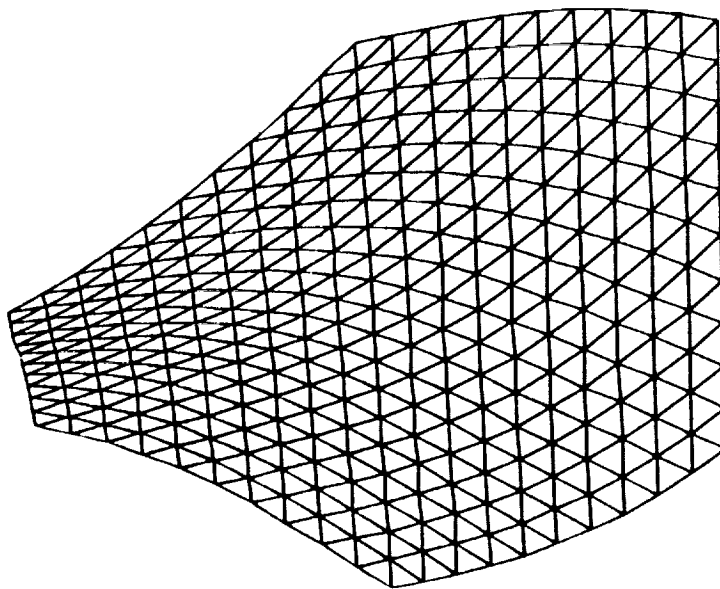
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Free Vibration of Hexagonal Panels Simply Supported at Discrete Points

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INTRODUCTION

Several deep-space science and planetary exploration missions currently being considered (ref. 1) will require using large hexagonal panels with some degree of curvature attached to a truss structure. Potential applications of these doubly curved faceted surfaces include a near-optical quality reflector surface with active shape control for a deep-space astrophysics facility, shown in figure 1, or a thermal protection system for a Mars-mission aerobrake. These two examples represent extremes in the application of these structures. Under operational conditions, the reflector panels would be subjected to relatively benign loads (e.g. station-keeping and pointing maneuvers), while maintaining an extremely accurate surface for the reflector to perform its mission. The aerobrake, on the other hand, may experience deceleration loads of up to 6 g's (ref. 2), but would not require active shape control. This study examines the natural vibration of a simply supported hexagonal panel using finite element methods. Two variables which have a significant influence on the panel behavior are examined. These variables are the panel radius of curvature and the location of the panel supports. In addition, the effect of using a panel support which restricts both normal and circumferential translations is also discussed. Because this structure is unloaded, this panel would be representative of the precision reflector segments described earlier, rather than a heavily loaded aerobrake surface.

PANEL DESCRIPTION

The structure examined in this report is a regular hexagonal surface, shown in figure 2, of edge length a , diameter across corners $2a$ and uniform thickness h . The panel has a doubly curved parabolic surface defined by:

$$z = \frac{1}{4f}(x^2 + y^2) \quad (1)$$

where x and y are the in-plane coordinates and f is the focal length of the assembled reflector. It is important to note that each panel in a segmented reflector would not have a symmetric cross-section, but rather a sector of a paraboloid of revolution. This is necessary to ensure that the assembled primary reflector has a parabolic surface. For the purposes of this study, it is assumed that the curvature in an individual panel can be modeled by equation 1. One panel examined in this study is simply supported at three points 120° apart on a circle of radius r , shown in figures 3 and 4(a). Each support point lies along a principal diameter of the panel and represents a discrete attachment point where the panel is attached to the support truss structure. A second panel attachment scheme studied locates the support points at six points on alternating radii of $r/a = 0.5$ and 1.0 . A planform view of this concept is shown in figure 4(b).

ASSUMPTIONS AND PARAMETERS

The natural frequency for a flat hexagonal panel is given by (ref. 3):

$$\omega = \frac{\alpha}{a^2} \sqrt{\frac{d}{\rho}} \text{ rad/s} \quad (2)$$

where the value of α , the nondimensional frequency parameter, depends on the boundary conditions. For a parabolic panel, this parameter will also be shown to depend on f , the reflector focal length. The other terms in equation 2 are a , the panel edge length, d , the bending stiffness of the panel (as defined in ref. 3) and ρ , the areal mass density of the panel. The panel planform and cross-sectional dimensions and material properties were assumed to be constant for the purposes of this study. The panel is assumed to be made of a linear, isotropic material and to behave according to Kirchhoff plate theory. In addition, the effects of structural damping and material hysteresis are neglected in these analyses. Thus, once the frequency parameter has been determined for a specific set of boundary conditions, the natural frequency may be estimated for any hexagonal panel of given edge length, thickness and material properties. For example, a panel edge length of 40 inches and thickness of 2 inches is representative of the

panel sizes being considered for large segmented reflectors (ref. 4). Using these dimensions, the natural frequency divided by the frequency parameter for a solid aluminum panel is 74.36 rad/s. The material properties assumed for aluminum were an elastic modulus of 10.0×10^6 lbf/in², Poisson's ratio of 0.3 and density of 2.59×10^{-4} lbf-s²/in⁴. Use of lightweight, high-stiffness composite materials would significantly increase the panel natural frequency without affecting the frequency parameter.

It is important to understand how the location of the support points affects the vibrational response of the individual panels. To achieve this, the panel was supported at nine possible values of r/a , the non-dimensional support location, between 0.2 and 1.0. This type of local behavior is of interest because low-frequency vibration of the panels could degrade the overall telescope performance by distorting the reflective surface. Unless the panel fundamental frequency is sufficiently high, this mode may be excited by inputs from the spacecraft's attitude control system. The hardware used to attach the panels to the support truss would provide some rotational stiffness at the support points and would also permit in-plane vibration of the panel. Thus, the simply supported boundary conditions (i.e., all three translational degrees of freedom restricted) applied provide a conservative lower bound on the panel's dynamic behavior.

For a fixed hexagonal truss planform, two distinct panel sizes which eliminate gaps between the panel edges have edge lengths which are 0.58 and 1.15 times the truss member length. These panel arrangements are shown in figures 4(a) and 4(b) respectively, with the support point locations shown as white circles. Examination of these figures leads to the conclusion that support locations of $r/a = 1.0$ are usable for attachment of both panel sizes, while attachments at $r/a = 0.5$ are possible for the larger panel size.

In addition, a hybrid six-point attachment scheme, shown in figure 4(b), may be used on the larger panels. In this scheme, the panel-to-truss attachments are made at points on alternating radii of $r/a = 0.5$ and 1.0. One potential drawback of the six-support-point concept is that it would greatly complicate active shape control of an aggregate reflector surface because the three additional support points doubles the number of actuators required and complicates the control algorithm for rigid-body positioning of each panel. In addition, since six support

points overdefine the plane of each reflector panel, residual forces may be introduced into the panels and support truss as a result of the statically indeterminate constraints. A simpler panel attachment scheme would use only three support points, either at $r/a = 0.5$ or 1.0 . Since these three points explicitly define a plane, rigid-body attitude changes of the panel could be implemented by extension of any of the three actuators shown in figure 5. With these considerations in mind, the natural frequencies and mode shapes of panels with these support arrangements will be computed and discussed.

The effect of curvature on the vibrational behavior of the panel will be studied for several values of f , the focal length of the primary reflector, representative of reflector focal lengths for systems currently under investigation (ref. 4). The reflector diameter, D , is assumed to be 787.4 inches (20m). The focal ratio, f/D , is defined as the ratio of the reflector's focal length to its diameter. The panel behavior for $f/D = 0.5, 1.0, 1.5$ and infinity (flat panel) were examined.

ANALYSIS AND RESULTS

Normal modes and frequencies were computed for the hexagonal panel using EISI-Engineering Analysis Language (EAL, ref. 5) for the range of parameters discussed above. A perspective view of the finite element model is shown in figure 6. The dark lines are the radii along which the supports are located. Each edge of the panel was modeled with 10 equilateral triangle plate bending elements (shell elements for the parabolic surfaces), for a total of 600 elements in the structure. The panel mass distribution was modeled using a consistent mass formulation in EAL. The supports restricted all three translational degrees of freedom at a support node (except as noted below) while permitting the three rotational degrees of freedom.

The lowest two modes of the flat panel (having a focal ratio of infinity) were computed for the range of support conditions described above. The computed natural frequency for each mode was divided by the geometric and material constants in equation 2 to determine the associated frequency parameter. These frequency parameters are shown as functions of the support point location for the flat panel in figure 7. The vertical dashed lines indicate two feasible values of the support point locations discussed earlier. The mode shape shown in figure 8 has

zero nodal diameters (radial lines of zero displacement) and one nodal contour (a band of zero displacement), shown as a darker line in the figure. The triangular nodal contour in this figure results from the boundary conditions. Using the notation of ref. 3, this is a $(0, 1)$ mode and is similar to the deformation pattern resulting from a gravity load normal to the panel. The resulting degradation in surface accuracy from the $(0, 1)$ mode is a defocus effect, where the actual focal point moves along a line containing the panel centroid and the ideal focal point. The frequency parameter for this mode is shown as a solid line in figure 7. The frequency parameter for the $(0, 1)$ mode increases to a maximum value of 9.59 at $r/a = 0.6$, which is close to the frequency parameter of the $(0, 1)$ mode for a completely free circular plate (ref. 3).

The other mode shape studied has one nodal diameter and zero nodal contours and is referred to as a $(1, 0)$ mode. This mode shape is asymmetric with respect to the nodal diameter and is shown from two viewpoints in figure 9. The darker lines in the figure represent the nodal diameters. Vibration in this mode will result in an astigmatic surface error in the panel, where the reflected radiation does not converge to a common point. In addition, this is a repeated mode, where the natural frequencies are repeated and the nodal line is rotated about the z -axis. Repeated modes result from multiple planes of symmetry in the basic structure. The frequency parameter for the $(1, 0)$ modes is shown as a dashed line in figure 7 and is relatively insensitive to the support point location, with a maximum difference of only 28.2 percent over the range of supports studied. The data shown in table 1 are the lowest of the two frequency parameters computed. The fundamental frequency parameter increases from a minimum value of 3.17, associated with the $(0, 1)$ mode when the panel is supported on the perimeter, to a maximum value of 5.02 (at $r/a = 0.4$) and decreases to a value of 4.56 at $r/a = 0.2$. The $(1, 0)$ modes become the fundamental behavior for all values of $r/a \leq 0.9$. Only the fundamental frequency parameters were reported for the parabolic panels.

Fundamental frequency parameters of the three parabolic panel models were computed for the same range of support locations as the flat panel and are listed in table 1. The results of these analyses also are presented as functions of the radial support location in figure 10. The limiting values from the baseline case described above are shown as a dashed line in this plot. The frequency

parameters for the parabolic panels are all higher than those for the flat panel, differing by a maximum value of 8.2 percent from the flat panel values for the $f/D = 0.5$ panel supported at the perimeter. The mode shapes for the parabolic panels are identical to those for the baseline case described above, with the (0, 1) mode occurring at an r/a of 1.0 and the (1, 0) modes occurring for the other support positions. The parenthetical data in table 1 are the fundamental frequency parameters for the parabolic panels computed using a boundary condition which restricts both normal and circumferential translations while allowing radial motion. Use of this type of support had the greatest effect on the (0, 1) mode frequency parameters, while the (1, 0) mode parameters were virtually identical. Results computed for the flat panel with unconstrained radial motion were identical to the data presented in table 1.

The fundamental frequency parameter and mode shape were also computed for panels having six pinned support points arranged as shown in figure 4(b), and the same range of focal ratios studied earlier. The computed frequency parameters are shown at the bottom of table 1. They range from 11.60 for a flat panel to 11.91 for a panel with $f/D = 0.5$, for a maximum difference of 2.7 percent. The associated fundamental mode shape is shown in figure 11. It has zero lateral displacement at each of the three vertices and midpoints of the inscribed equilateral triangle, where the supports are located. The panel motion inside the triangle is out of phase with that outside the triangle. This mode shape is similar to the (0, 1) mode shape discussed earlier, except that the zero-displacement boundary (shown as a darker line in the figure) is an equilateral triangle. This attachment scheme offers a significant increase in the panel fundamental frequency over the three-point methods presented. However, other considerations, some of which were discussed, may preclude operational use of this concept.

Higher frequencies occur for the parabolic panels because the bending and in-plane stiffnesses are coupled in doubly curved structures, resulting in a higher flexural stiffness. This stiffening effect becomes more pronounced as f/D decreases. Analytical treatment of a similar problem (ref. 6) adds a term involving the in-plane stiffness and panel curvature to the square of the flat-panel natural frequency to obtain the square of the natural frequency of the doubly-curved panel. It was also observed that the parabolic panel frequencies approach the

flat panel values for support points near the panel center. The time required to compute frequencies for the doubly-curved panels was significantly higher than that required for the flat panel cases. The additional computation time resulted from the introduction of the additional degrees of freedom required to model a three-dimensional shell structure.

MODEL VERIFICATION

As with any other finite element solution, the accuracy of the computed results should be verified by classical (or, if possible, experimental) techniques. Since no theoretical results exist for the specific problem under study, a similar structure which has an analytical solution was studied to evaluate the accuracy of the computational model. The theoretical frequency parameter for a flat hexagonal plate with simple supports on all six edges was found to be either 6.96 or 7.13 (using two different methods; ref. 3) for the fundamental mode. A finite element model of this structure was generated in EAL. Simple support of the panel edges was approximated by boundary conditions which restricted the three translational degrees of freedom along the edges, but left the rotational degrees of freedom unconstrained. The computed frequency parameter for this structure was found to be 7.12, for an error of 2.30 and 0.14 percent (respectively) compared to the theoretical solution. Based on these results, this model should yield equally accurate results for the problem of interest.

CONCLUSIONS

The fundamental frequency for hexagonal panels of varying degrees of curvature, simply supported at three discrete points, was calculated using finite element methods. Observed trends in the computed frequency parameters indicate that, for the values of focal ratio and support location studied, the flat panel frequencies provide a close approximation to the fundamental frequency of the doubly curved panel. The baseline case of a flat panel is conservative because it provides a lower bound on the frequency response of the parabolic panels. The panel support location was seen to have a large effect on the frequency parameter of the (0, 1) mode, while the (1, 0) asymmetric mode frequency parameter was less sensitive to the choice of support location. These considerations are important in the design process, where a reasonably accurate

estimate of the component behavior is required, as well as an appreciation of the effect of component-level behavior on the overall performance of the system.

Additionally, the structural dynamic behavior of a hybrid six-point support scheme was examined. This panel had a frequency parameter which was over twice the highest fundamental parameter of the three-point support concepts and showed a small variation for the range of panel curvatures studied. The fundamental mode shape was also seen to resemble that of the (0, 1) mode.

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Table 1. Frequency parameters for fundamental mode of a simply supported hexagonal panel (Radially free supports shown parenthetically)

r/a	$f/D=\infty$	$f/D=1.5$	$f/D=1.0$	$f/D=0.5$	Mode
0.2	4.56	4.57 (4.57)	4.58 (4.57)	4.59 (4.58)	(1, 0)
0.3	4.98	4.99 (4.98)	4.99 (4.98)	5.00 (4.99)	(1, 0)
0.4	5.02	5.03 (5.02)	5.03 (5.02)	5.05 (5.02)	(1, 0)
0.5	4.91	4.92 (4.91)	4.93 (4.91)	4.96 (4.91)	(1, 0)
0.6	4.73	4.76 (4.73)	4.78 (4.73)	4.82 (4.74)	(1, 0)
0.7	4.53	4.57 (4.53)	4.59 (4.53)	4.65 (4.53)	(1, 0)
0.8	4.31	4.36 (4.31)	4.39 (4.31)	4.47 (4.31)	(1, 0)
0.9	4.06	4.12 (4.06)	4.15 (4.06)	4.24 (4.06)	(1, 0)
1.0	3.17	3.26 (3.19)	3.30 (3.20)	3.43 (3.22)	(0, 1)
hybrid 6-point	11.60	11.71	11.76	11.91	(0, 1)

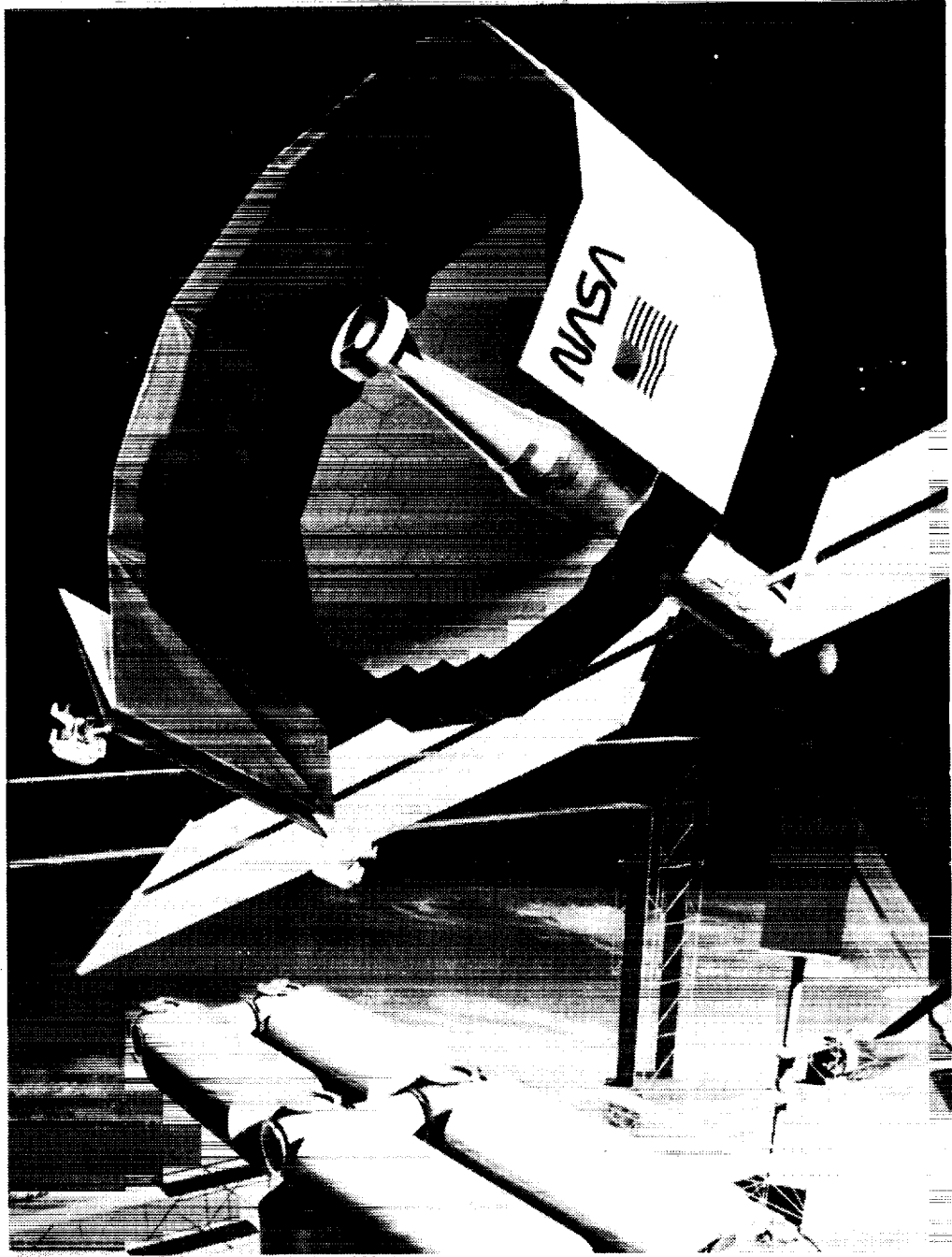


Figure 1. 20 meter-diameter submillimeter astronomical observatory

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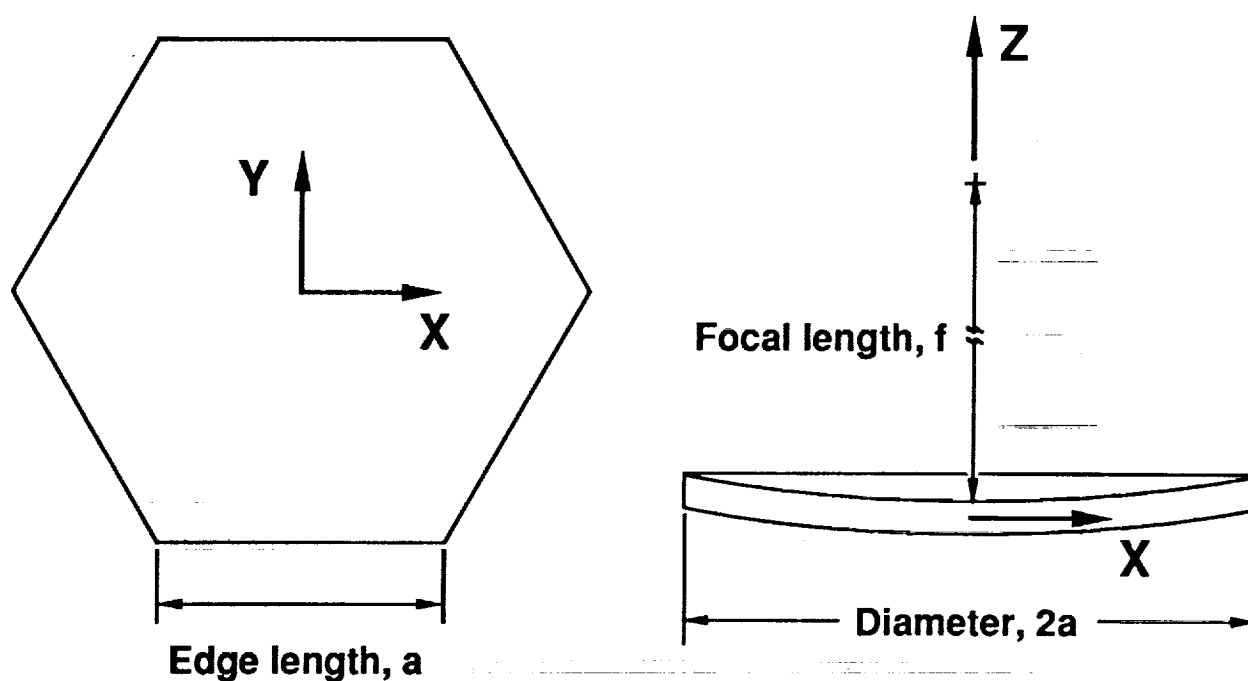


Figure 2. Hexagonal panel geometry

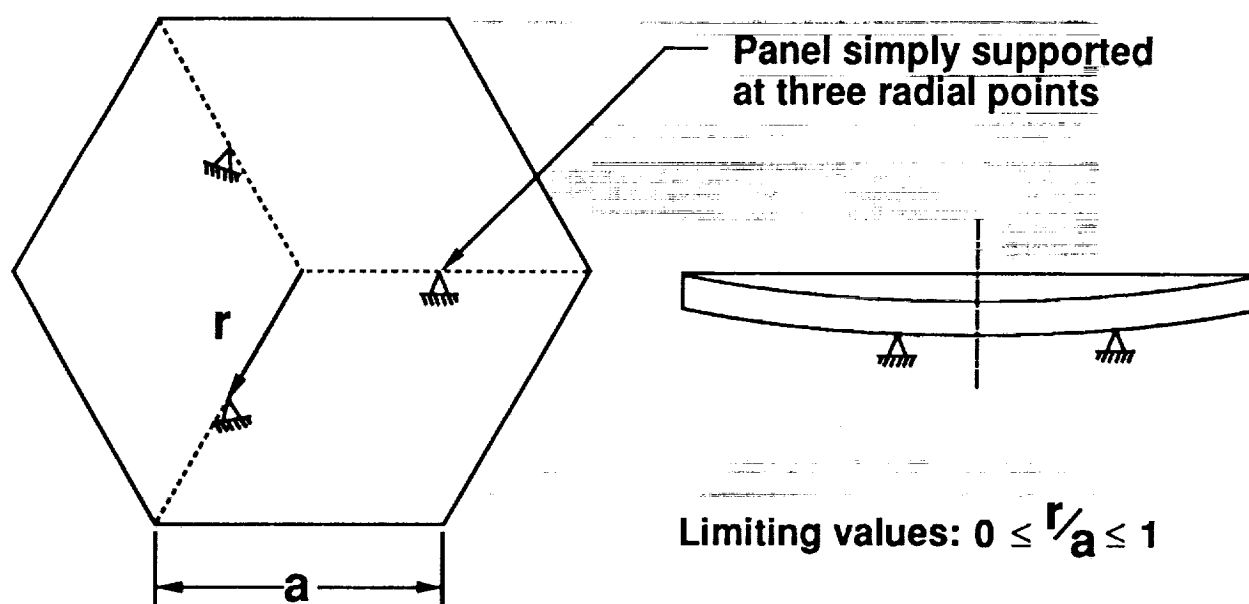
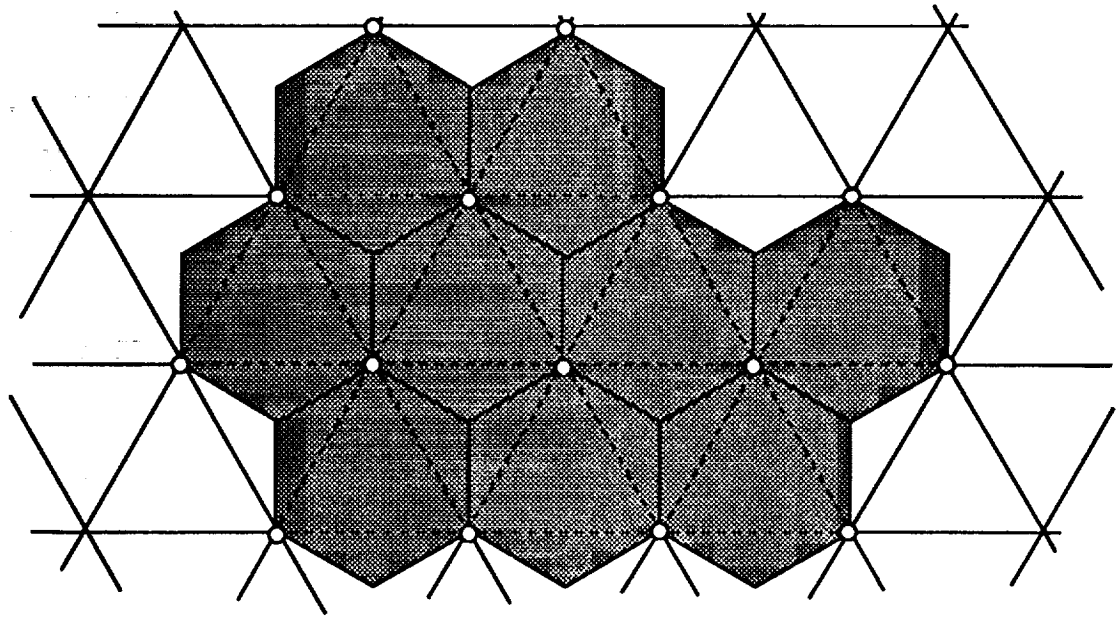
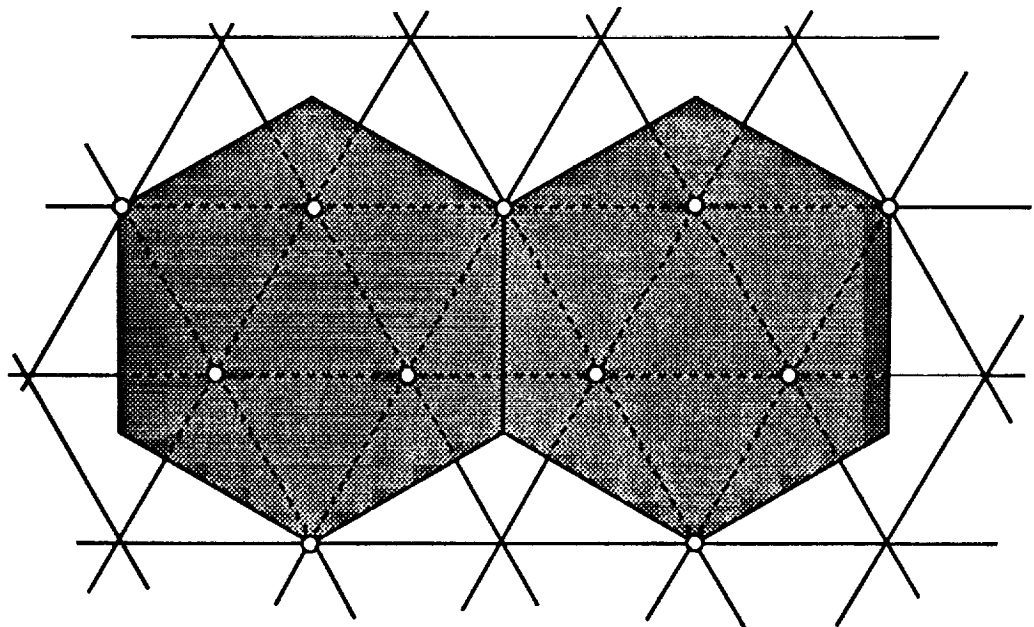


Figure 3. Panel support conditions



(a) Small hex panel mosaic



(b) Large hex panel mosaic

Figure 4. Arrangement of panels to form reflective surface

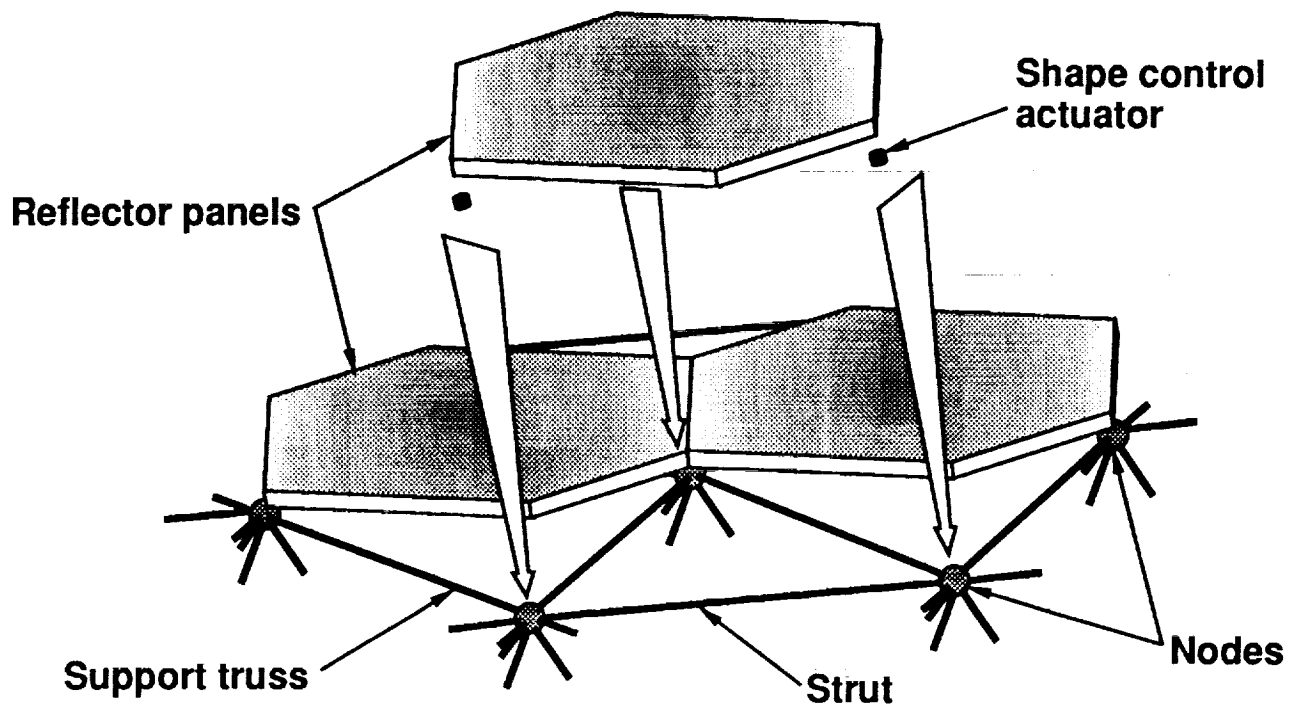


Figure 5. Panel attachment schematic

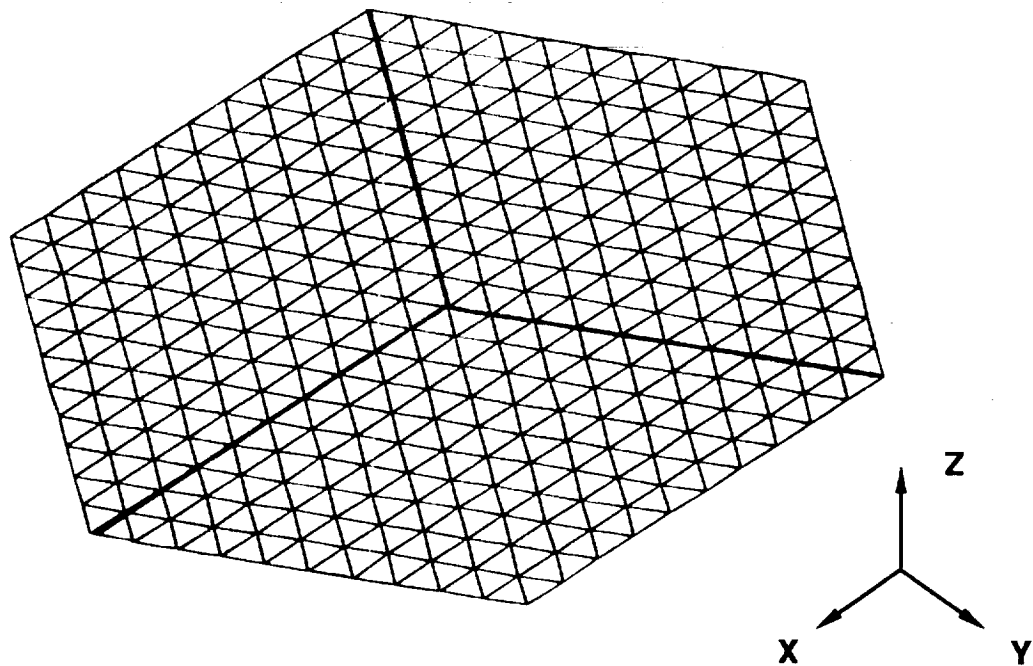


Figure 6. Panel finite element model

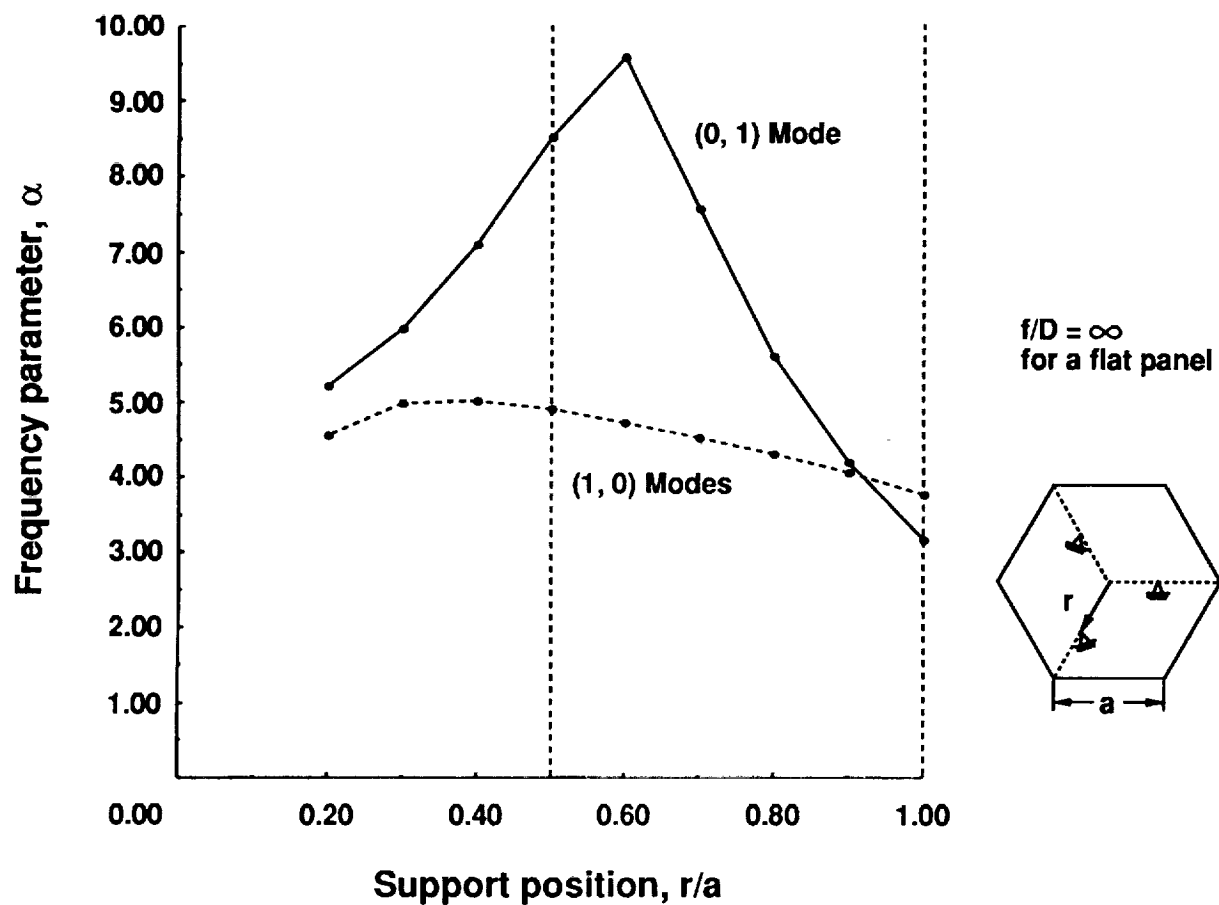


Figure 7. Frequency parameter for lowest two modes of a flat hexagonal panel

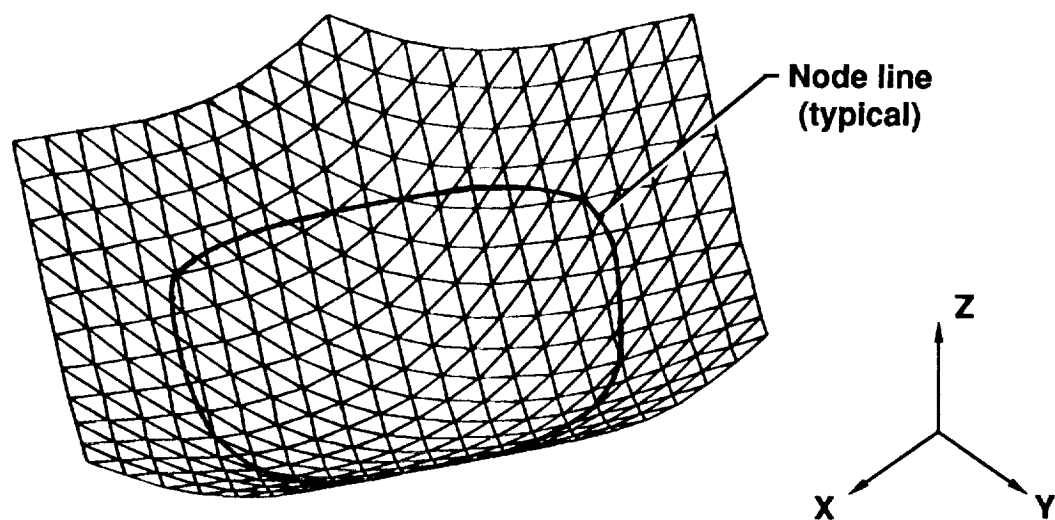


Figure 8. (0, 1) mode shape for a flat hexagonal panel

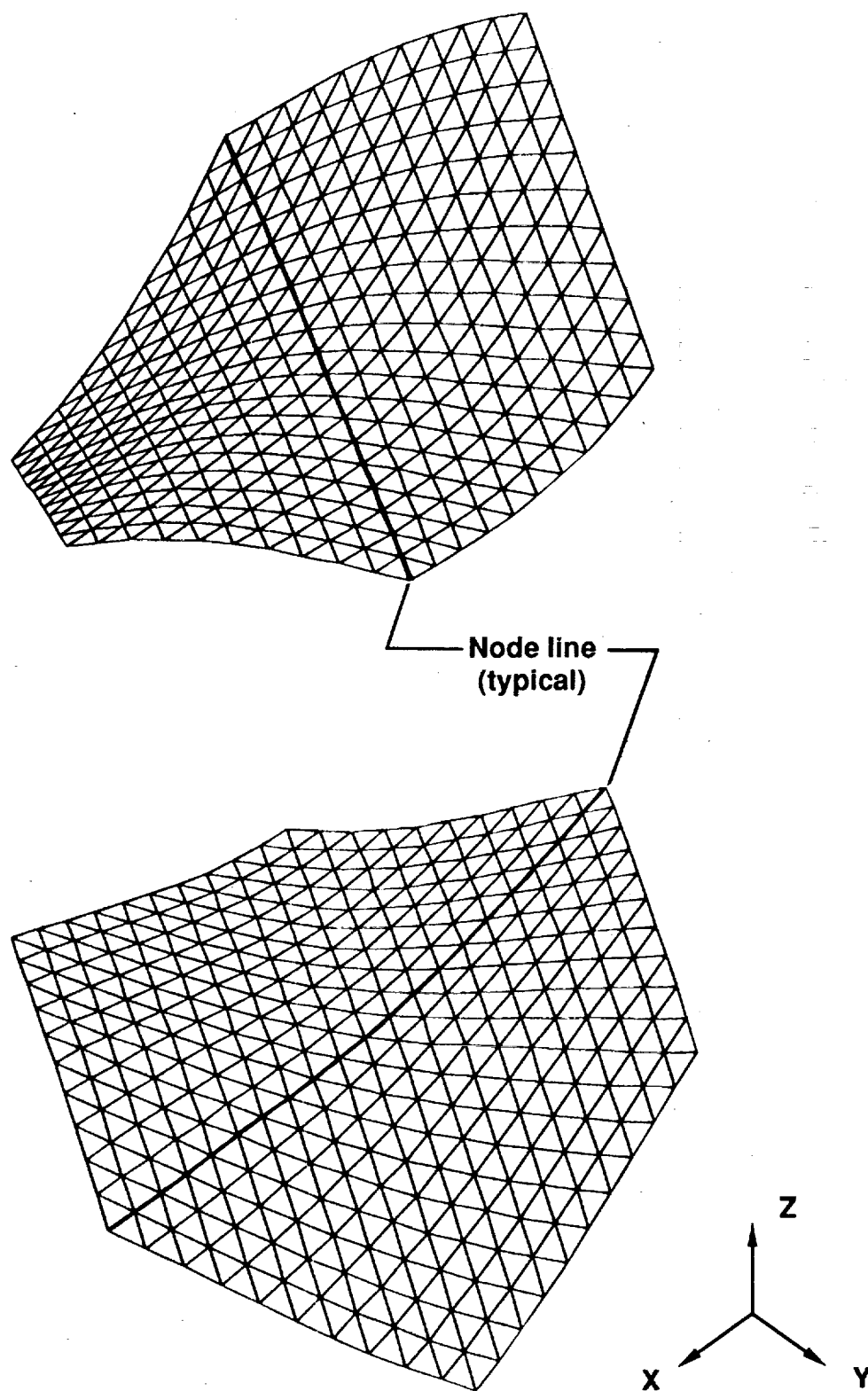


Figure 9. (1, 0) mode shapes for a flat hexagonal panel

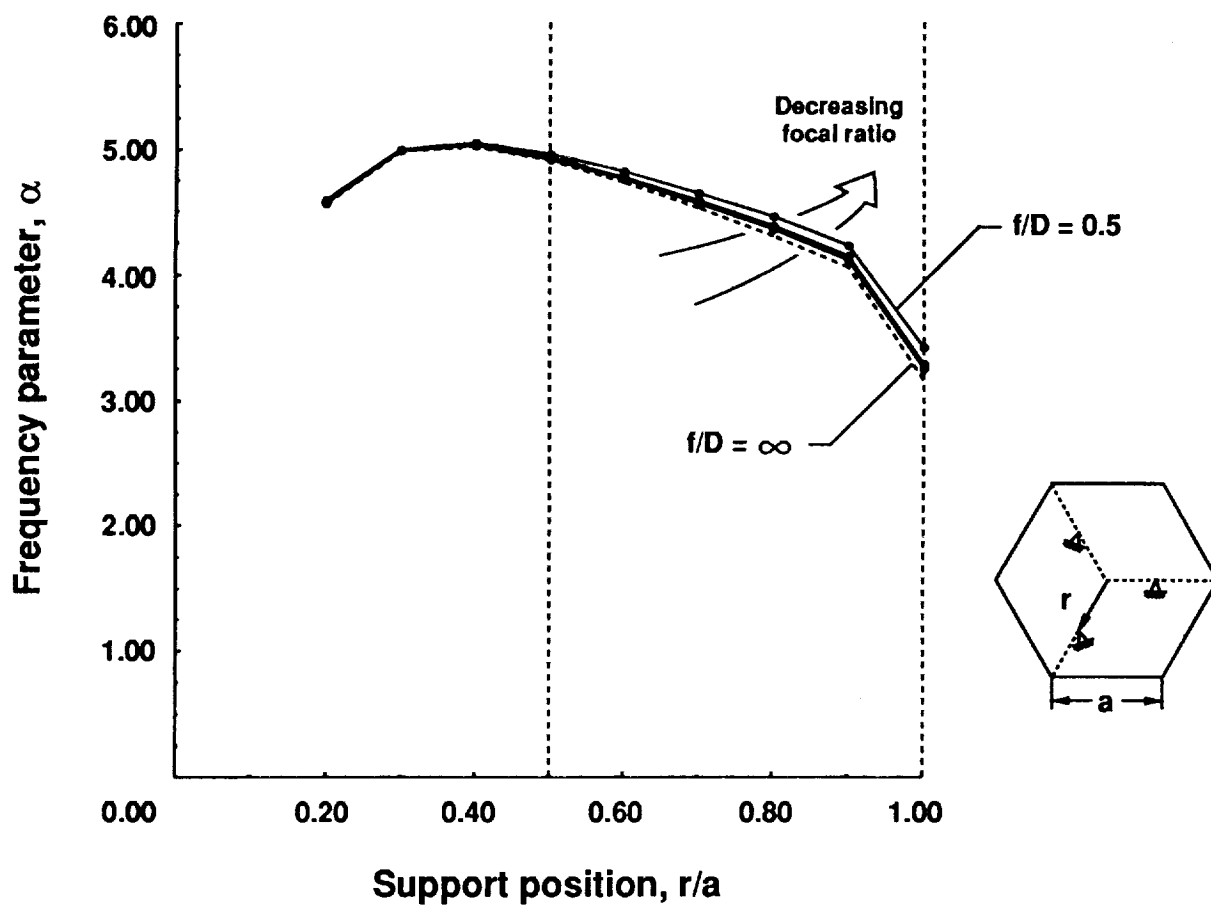


Figure 10. Fundamental frequency parameter for parabolic hexagonal panels

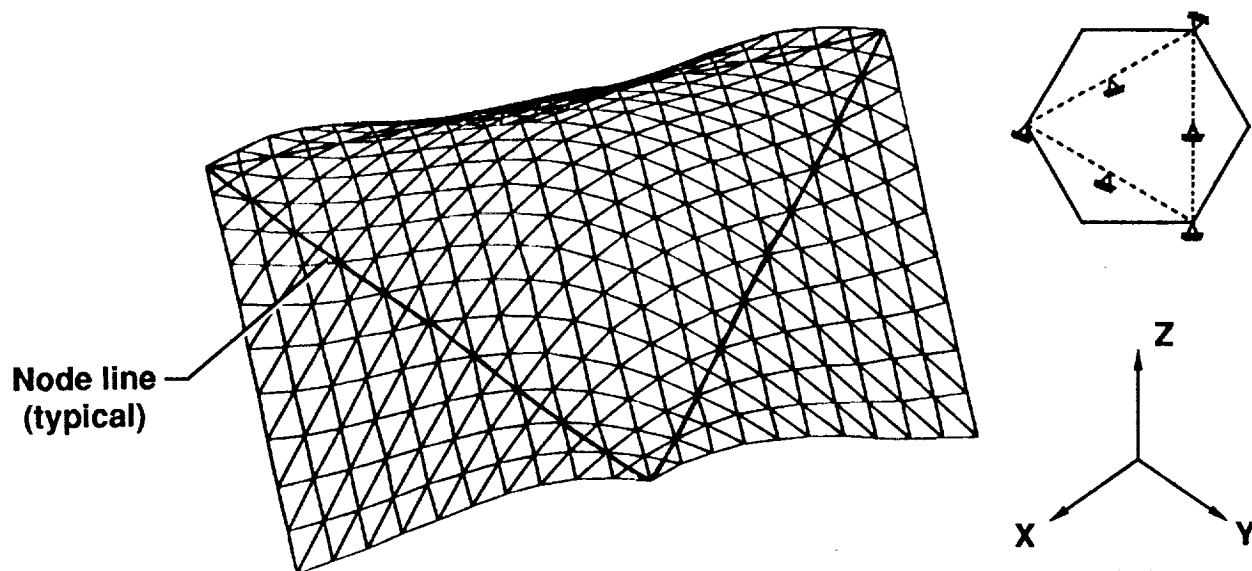


Figure 11. Fundamental mode shape for a hexagonal panel supported by a six-point hybrid attachment concept



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16. Abstract An analytical study to determine the structural dynamic behavior of a hexagonal panel with discrete simple supports is presented. These panels are representative of the facets of a precision reflector surface. The effects of both support point location and panel curvature on the lowest natural frequency of the panel are quantified and discussed. The panels are simply supported at three equally spaced points. The panel curvature is modeled as a parabola having the same focal length as the assembled reflector. A flat panel and three representative values of panel curvature are studied. The natural frequency is composed of a constant which depends on the panel planform, thickness and material properties as well as a non-dimensional frequency parameter which depends on the support conditions and panel curvature. The frequency parameter is computed for each of nine support point locations and a given panel curvature. The frequencies for a flat panel are shown to provide a conservative lower bound on the curved panel frequencies for all values of the support point locations and panel curvatures. In addition, the frequency parameters are relatively insensitive to the support location over the range of values studied. The frequency parameters for a six-point support concept is also studied for the same panel curvatures. The fundamental frequency is higher than the panels supported at three points, but active control of this concept may prove difficult.					
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